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It may be added that this gives a value for the absolute semi-diameter of Moon equal to

$$15\ 33''\cdot 36,$$

which is in close accord with the value

$$15'\ 33''\cdot 35,$$

deduced by myself from the 1500 observations of the Moon made at Greenwich between 1862 and 1877, and with the value

$$15\ 33\cdot 32$$

deduced by Prof. Winnecke of Strasbourg.

I would add that this correction removes a difficulty which has long perplexed me—namely, a sudden alteration in the apparent value of the mean semi-diameter of the Moon at the epoch 1862, the observed values prior and subsequent to that date seeming to differ by rather less than a second of arc. This discordance is referred to in several of my earlier papers, written six or seven years ago, when I was unable to refer to other of Hansen's publications than his Tables.

On the Determination of the Coefficient of the Parallactic Inequality in the expression for the Moon's Longitude. By E. J. Stone, M.A., F.R.S.

In the *Monthly Notices* for 1880, May and June, there appears a paper by Messrs. Campbell and Neison "on the Determination of the Solar Parallax by means of the Parallactic Inequality in the Motion of the Moon."

This paper contains some criticism, to which I do not object, of a paper of mine on the same subject printed in 1867; it contains also some erroneous corrections to the result of that paper to which I must object, and raises some points of interest which require examination.

The direct results of Messrs. Campbell and Neison's paper are as follows:—

If we select from the observations of the Moon made at Greenwich, with the Transit-Circle, during the years 1862 to 1876, both inclusive, those which were taken when the Moon passed within two days of the first and third Quarter, and throw the discordances between the mean results for errors of Hansen's Tables upon the Parallactic Inequality, the coefficient, which best satisfies the observations with a *semi-diameter adopted from the time of passage of the diameter over the meridian when near the Full Moon*, is 125″·06.

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The results are very fairly accordant, and the method is the one which has been previously generally used. This value of the coefficient of the parallactic inequality, 125″·06, would correspond to a value of the solar parallax of about 8″·81.

The coefficient of the parallactic inequality, thus determined, is, however, systematically too small by the amount of the mean excess of the apparent semi-diameters near the Full over those near the two Quarters. The value of the solar parallax 8″·81 is for the same reason too small by about one-fourteenth part of the mean excess of the semi-diameter near the Full, and it can therefore only be regarded as an inferior limiting value of the solar parallax. But although this is clear enough, there are great practical difficulties in investigating the *exact* amount by which the coefficient of the parallactic inequality thus determined should be increased. We know that the required increase cannot exceed 2″, and it most probably does not amount to quite half that quantity; this fixes, with the value of the coefficient 125″·06, which is itself of course liable to error, a superior limit to the value of the solar parallax of about 8″·96, and an inferior one of about 8″·81.

Messrs. Campbell and Neison have attempted to correct for the effects of irradiation as follows:—

They assume semi-diameter near the Quarters

= semidiameter near Full Moon − 0·25I for the winter observations,

= semidiameter near Full Moon − 0·75I for the summer observations,

and determine the quantity I by equating the values of the coefficient of the parallactic inequality which is derived from the discussion of the summer and winter observations. The result thus obtained for the coefficient of the parallactic inequality, 125″·66, is so far satisfactory that it gives an increase of about 0″·6 for the mean excess of semi-diameter near the Full over that of the Quarters, a quantity which from general considerations might be assumed as an approximation to the truth.

The value 125″·66 of the coefficient of the parallactic inequality corresponds to a value of about 8″·85 of the solar parallax. But the weak point in this argument is the assumption of the coefficients 0·25 and 0·75 of the unknown quantity I in the winter and summer equations of condition. It is not stated *how it is found* that these coefficients are approximately correct. Unless it can be shown that there are no admissible variations in the ratio of these assumptions sufficiently important to change the deduced value of the coefficient of the parallactic inequality, the assumptions themselves are not legitimate; and, although the step may be disguised, the investigation is as little satisfactory as the direct assumption, from general considerations, of the irradiational enlargement.

The following table of the values of I for the different years is given by Messrs. Campbell and Neison:—

Year.	Value I.
1876–1875	I = +11″·92
1875–1874	= − 2·76
1874–1873	= − 7·96
1873–1872	= − 3·61
1872–1871	= − 5·22
1871–1870	= − 4·75
1870–1869	= − 0·56
1869–1868	= + 4·35
1868–1867	= + 5·06
1867–1866	= − 4·03
1866–1865	= − 1·88
1865–1864	= − 1·68
1864–1863	= − 0·74
1863–1862	= − 2·22

Upon these discordant results the authors remark as follows:—“It is obvious that any results based on such wild values must be fallacious. It shows the misleading character of values obtained from the means of large numbers of observations without examining the separate results to see if they are really accordant. The result of the preceding investigation might be regarded as discouraging, and as showing that it would not be practicable to determine in this manner the effect of the variation in the amount of irradiation at the limb of the Moon. If this were really the case, it would be most unsatisfactory, and throw grave doubt on the possibility of determining the true value of the parallactic inequality from observations of the transit of the lunar limb. For the only permissible method of ascertaining the amount of this variation in the irradiation is by this process of comparing the summer with the winter observations. If this prove to be impracticable, it involves as a necessary consequence the failure of the whole method of determining the solar parallax from these observations.

“Further consideration shows that these results are too discordant for this view to be adopted. These extreme discordances can only arise from systematic errors due to imperfections in the tables, or method of reducing the observations. These discordances must be due to the failure of the assumption which we have adopted, in common with Mr. Stone and Prof. Newcomb, and taken as the basis of our work—the assumption that in the mean of a number of observations all errors except those of the parallactic inequality can be neglected and be trusted to destroy one another.”

I may mention that these remarks are not applicable to my work. I, at least, did not assume that the errors of any existing lunar tables would mutually destroy each other's effects in obser-

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vations spread over half a year. But, waiving this point, which is not important, I pass to the consideration of what would appear to be the chief result of Messrs. Campbell and Neison's paper.

To avoid the unsatisfactory results found by the method already explained, the authors introduce into the equations of condition "twelve unknown quantities, or rather variable corrections," but the *same fundamental assumptions are made with respect to the irradiation.*

The value of the coefficient of the parallactic inequality thus found is 125″·64, a result almost identical with that obtained in the second case without the introduction of the additional corrections. But the importance of the additional corrections appears to me much over-estimated, and to test this point I have calculated, from the data of the paper, the values of I for the different years, and have obtained the following residual errors:—

	Residuals from Incomplete Solution.	Residuals from Complete Solution.
	″	″
1876–1875	+12·60	+9·43
1875–1874	− 2·08	−2·34
1874–1873	− 7·28	−5·94
1873–1872	− 2·93	−1·16
1872–1871	− 4·54	−0·88
1871–1870	− 4·07	−1·11
1870–1869	+ 0·12	+5·28
1869–1868	+ 5·03	+8·68
1868–1867	+ 5·74	+6·85
1867–1866	− 3·35	−2·29
1866–1865	− 1·20	−1·66
1865–1864	− 1·00	−2·50
1864–1863	− 0·06	−3·33
1863–1862	− 0·54	−0·14

It appears to me that the residuals from the general investigation are hardly less unsatisfactory than those previously obtained, and that the introduction of these "twelve unknown quantities, or variable coefficients," and the use of methods with some pretensions to refinement and precision, have not led to a much more satisfactory result than the simpler investigation which has been so severely condemned.

The probable error attached by the authors to the result 125″·64 is only +0″·09, and the value of the solar parallax is 8″·848±0·007.

It is inconceivable to me how such probable errors have been obtained.

To show that I have not objected to the method adopted by Messrs. Campbell and Neison for the determination of the irra-

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diation without some grounds, I give the following numerical illustration of my objection.

Messrs. Campbell and Neison find I from the equations

$$(1)\quad m\mathrm{I}+d\mathrm{P} = -2''\!\cdot\!00$$
$$(2)\quad n\mathrm{I}+d\mathrm{P} = -1\cdot40,$$

which give

$$\mathrm{I} = -\frac{0'\!\cdot\!60}{m-n}$$

and

$$d\mathrm{P} = -1''\!\cdot\!70+\frac{m+n}{m-n}\,0''\!\cdot\!30,$$

or

$$\mathrm{P} = 125''\!\cdot\!06+\frac{\frac{m}{n}+1}{\frac{m}{n}-1}\,0''\!\cdot\!30.$$

The quantity 125″·06 is derived from the general investigation and not strictly from (1) and (2) or from the adopted value minus 1″·70.

Now, with $m = 0\cdot75$, and $n = 0\cdot25$, this gives

$$\mathrm{P} = 125''\!\cdot\!66.$$

But supposing we take $m = 0\cdot8$, and give to n the values

0·2, 0·3, 0·4, 0·5, and 0·6

respectively, we should then obtain for P the corresponding values

125″·56, 125″·72, 125″·96, 126″·36, and 127″·16.

In fact, we can select a value of the solar parallax and make this result apparently come from a discussion of the observations by determining the ratio $m : n$ from the equation

$$\frac{\frac{m}{n}+1}{\frac{m}{n}-1}\,0''\!\cdot\!3 = \delta\mathrm{P},$$

where $d\mathrm{P}$ is the correction required by P to give the selected value of the solar parallax.

I want, therefore, an independent proof that $m = 0\cdot75$ and $n = 0\cdot25$ are close approximations to the true values of m and n before I can attach any importance to the correction thus obtained for irradiation.

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I now pass to what, if true, would be an important result in Messrs. Campbell and Neison's paper. From the values of the coefficient of the parallactic inequality obtained by the Astronomer Royal in a discussion of the Greenwich Observations 1754 to 1843, Messrs. Campbell and Neison have inferred the existence of an inequality in the Moon's longitude $= 1''\cdot20 \sin \{D - 8\,(y - 1825\cdot5)\}$, where D is = mean long. of Moon − mean long. of Sun, and they have shown that such an inequality, if it existed, would change their value of the coefficient of the parallactic inequality from $125''\cdot64$ to $124''\cdot64$, and value of the solar parallax from $8''\cdot85$ to $8''\cdot78$.

No attempt is made to *prove* the *necessary existence* of such an inequality; and although, of course, its existence can be proved by its satisfying a sufficient number of the results of observation, yet if it rests simply upon this basis it is absolutely necessary that the results, derived from the assumption of its truth, should agree, not only with the old imperfect observations from which it was derived, but also with the modern observations which have been made with greatly improved instrumental means.

That it does not satisfy the results of observation has not altogether escaped the notice of the authors, for on page 405 we find:—"The computed value of the correction for the group 1843–1851 is $122''\cdot75$, whilst that actually found from the observations is $124''\cdot53$, there being a difference of $1''\cdot78$. Almost the entire portion of this difference arises from the high values found for the parallactic inequality from the observations made during the years 1846–1851, as those found for the earlier years were much lower."

But although it is thus perceived that their inequality gives nearly a minimum value where it should, according to the observations, have given a maximum value, the authors, apparently, were not prepared to face the fact; and they have attempted to escape the difficulty by reasoning which is certainly curious. They remark:—"It would, in fact, seem as if the apparent semi-diameter of the Moon had increased by $2''$ during these years, either from a change of observers or some other cause. This would have increased the apparent value of the parallactic inequality by $2''\cdot6$, and as this increase affects two-thirds of the observations, it would correspond to a fictitious increase of $1''\cdot73$, whilst that actually observed was $1''\cdot78$." I need hardly point out that this step is equivalent to a tacit assumption of the existence of the forty-five year period inequality, and that discordances between its results and those given by observations arise from errors of observations. Such assumptions are always unsafe, and are particularly unsafe in this case, when the only evidence in favour of the existence of the inequality consists of certain discordances between the results obtained by means of the imperfect instruments in use from 1750 to 1816; and when the discussion of these results is complicated by changes of instruments during the period, and the acknowledged

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impossibility of any satisfactory determination of the instrumental semi-diameter upon which so much depends.

In the year 1867, when the question whether the true value of the solar parallax was nearer 8″·58 or 8″·90 was an open one, I selected some 2,000 observations of the Moon made near the Quarters at Greenwich with the Altazimuth and Transit-Circle, to test, for my own satisfaction, the legitimacy of the proposed increase of the solar parallax. My method was the same as that followed by Messrs. Campbell and Neison, in section 1, and the result for the coefficient of the parallactic inequality was 125″·36. I had not the slightest expectation of fixing the coefficient within 0″·1, but I stated then and I believe now that the error cannot amount to more than 0″·4, and is probably much less. The semi-diameter used was that obtained from observations near the Full, and the value of the solar parallax thus deduced, about 8″·84, was therefore a minimum value of the result. This investigation appeared to entirely confirm the necessity of the proposed increase of the solar parallax. The result was briefly indicated in the *Monthly Notices* for Nov. 1867.

Messrs. Campbell and Neison profess their ignorance of the exact observations made use of by me, of the Tables through which I have compared the observations with theory, and of any systematic corrections which I may have applied to the results. Their position would therefore have been logically unassailable had they expressed themselves dissatisfied with the result on account of the impossibility of an examination of its freedom from systematic errors; but, instead of taking this course, they have proceeded to investigate the corrections which my result requires; and after thus reducing it from 125″·36 to 123″84, they appeal to the difference between this, supposed corrected result, and their own, as tending to strengthen the hypothesis of the real existence of the forty-five year period inequality.

Unfortunately for this argument, the corrections which they have applied to my result are certainly erroneous. The corrections which that result requires, if any, are small, and tend rather to increase than to decrease the resulting value of coefficient.

The observations made use by me in 1867 were:—

(1) The Altazimuth Observations, 1848–1861.

(2) The Altazimuth Observations, 1862–1866.

(3) The Transit-Circle Observations, 1853–1861.

(4) The Transit-Circle Observations, 1862–1865.

The observation in sections (1) and (3) were compared directly with Burckhardt's longitudes. Those in (2) and (4) were compared with Hansen's Tables.

After the discovery, by Professor Adams, of the errors in Burckhardt's expression for the parallax, the whole of the reduced

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observations were systematically corrected by the Astronomer Royal for errors of parallax and semi-diameter. It is these corrected results which were used by me. The semi-diameters were carefully discussed and separate determinations made for the two instruments, and the corrections required to the coefficient of the parallactic inequality on account of the errors in the coefficient of Burckhardt's variation, annual equation and *term* sin 3D were determined in mean results and applied.

As the results for the Altazimuth instrument extend now over more than thirty years, and are valuable, I give the separate results, extracted from my old papers, for the years 1848 to 1866, and have extended them to 1878 on the same principles. I profess here no attempt to secure greater accuracy than 0″·2.

The results 1848–1861 are derived from the errors of Burckhardt's longitudes after the observations have been reduced with Adams's parallax and a mean semi-diameter = 15′ 34″·52.

From a direct comparison between the values of the semi-diameters adopted by Adams and Hansen we have

Adams' = Hansen's + 0″·52.

The corrections to Hansen's semi-diameter now used at Greenwich are

+ 0″·49 for Azimuth Observations,
+ 1·01 for Altitude Observations.

My value agrees very closely indeed with that derived from the Azimuth Observations and, although it is slightly smaller than the mean result from the Azimuth and Altitude Observations, I appear to have investigated the necessary correction; and it is in my opinion doubtful whether it should be changed.

The correction + 1″·56 has been applied to the results obtained directly from the observations for the errors of the coefficients in Burckhardt's Tables. Almost the whole of this correction for the Altazimuth observations arises from error in coefficient of the term sin 3D.

	Value of P.		Value of P.
1848	″ 124·13	1855	″ 125·62
1849	123·52	1856	125·17
1850	125·70	1857	124·40
1851	126·18	1858	124·08
1852	126·45	1859	125·03
1853	126·98	1860	125·38
1854	127·50	1861	126·17

Mean value from 1848–1856 = 125″·69; 1833–1861, 125″·59.

These are derived through Burckhardt's Tables.

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From Hansen's Tables with semi-diameter corrected:—

	P.		P.
	″		″
1862	125·92	1871	124·90
1863	124·32	1872	124·80
1864	125·22	1873	125·25
1865	125·12	1874	125·68
1866	125·05	1875	123·36
1867	125·40	1876	125·35
1868	125·04	1877	125·44
1869	124·72	1878	125·81
1870	124·72		

It will be seen that the existence of the inequality proposed by Messrs. Neison and Campbell is not at all confirmed by these results. Instead of requiring a large positive correction about 1850–1855, the value of P is very large; and if it be considered that the results derived through Burckhardt's Tables are not to be trusted for the results of separate years, similar evidence against this assumed inequality is shown by the later results.

The observations about 1870 should give a maximum result, if the inequality really exists; but this is not shown to be the case in the Transit-Circle results collected by Messrs. Campbell and Neison, whilst for the Altazimuth observations we have:—

	Uncorrected.	With Neison's proposed Correction.	
	″	″	
P =	125·17	124·33	1862–1867
	124·84	113·66	1868–1872
	125·15	124·31	1873–1878

The greatest discordance between the uncorrected results is 0″·33, whilst this is increased to 0″·67 by the introduction of the correction proposed by Messrs. Campbell and Neison.

It would almost appear that, so far from this forty-five year period inequality satisfying the observations, its effects are nearly reversed, and give a maximum instead of a minimum about 1870, and a minimum instead of a maximum about 1848.

It is clear however to me, that the attempt to introduce this term is a retrograde step, and I have freely criticised the paper because I know how difficult it is to get rid of such results, when once they find their way into our *Notices*.

It appears to me clear that the value of the parallactic inequality does not differ greatly from 125″·23 + x, where x is some positive quantity which probably does not differ greatly from 0″·5, but is certainly less than 2″.

In this case the true value of the solar parallax is not very